computer, operating by counting electrical pulses. However, data can be input in either digital or analog form and the output can be of either type. Because of its moderate cost and this dual capability, the DDA will probably be used to a greater extent as time goes on.

That information on analog and digital computers might need to be compared was early recognized and much work has been done on design of analog-to-digital and digital-to-analog converters. In one type of the latter, digital values cause shaft rotations which are proportional to voltages, these being the analog representations of the digital quantities. A complete system consisting of a high-speed digital computer and a large analog computer, the two connected by converters, is under development at Ramo-Wooldridge. Thus the speed and accuracy of the digital computer will be augmented by the continuous recording of data of the analog computer, and the processes of data sampling and simulation can be studied more thoroughly.

CONCLUSION

Many problems in science, engineering, and business whose solutions once seemed to be beyond reach are now being successfully attacked by the use of high-speed electronic digital computers. Their use has made possible the construction of more complex mathematical models, with a resultant expansion and increase of our knowledge in many fields. In the business world, problems which demand an enormous number of elementary operations, such as inventory and payroll calculation, now are done in a small fraction of the time previously required. The automatic digital computer is also becoming the central element in control systems that are the basis of our progress toward automation.

Analog computers have demonstrated their usefulness in the continuous recording of data from wind-tunnel tests and similar problems. Their low cost and ease of construction are important considerations in making a choice as to the type of computer to be used.

The nonspecialist may have little or no direct contact with computers of any kind. Nevertheless, he should regard himself as a part-owner of such machines, because his Government, through purchase or rental, has invested large sums of money in them.

Pappus's extension of the Pythagorean Theorem

by Howard Eves, University of Maine, Orono, Maine

Every student of high school geometry sooner or later becomes familiar with the famous Pythagorean Theorem, which states that *in a right triangle the area of the square described on the hypotenuse is equal to the sum of the areas of the squares described on the two legs.* This theorem appears as Proposition 47 in Book I of Euclid's *Elements*, written about 300 B.C.

Even in Euclid's time, certain generalizations of the Pythagorean Theorem were known. For example, Proposition 31 of Book VI of the *Elements* states: *In a right triangle the area of a figure described on the hypotenuse is equal to the sum of the areas of similar figures similarly described on the two legs.* This generalization merely replaces the three squares on the three sides of the

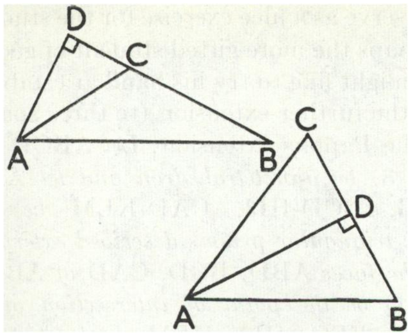


Figure 1

right triangle by any three similar and similarly described figures. A more worthy generalization stems from Propositions 12 and 13 of Book II. A combined and somewhat modernized statement of these two propositions is: *In a triangle, the square of the side opposite an obtuse (acute) angle is equal to the sum of the squares on the other two sides increased (decreased) by twice the product of one of these sides and the projection of the other side on it.* That is, in the notation of Figure 1, $(AB)^2=(BC)^2+(CA)^2\pm 2(BC)(DC)$, the plus or minus sign being taken according as angle C of triangle ABC is obtuse or acute.

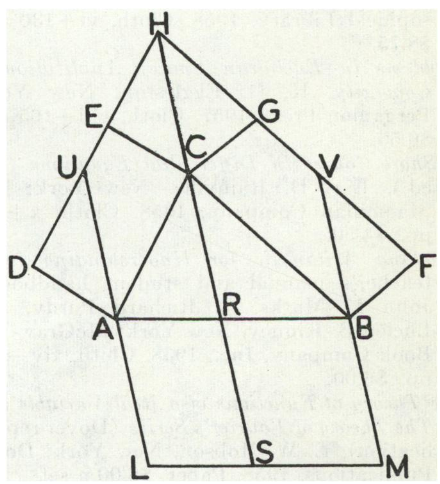
If we employ directed line segments we may combine Propositions 12 and 13 of Book II and Proposition 47 of Book I into the single statement: *If in triangle* ABC, D *is the foot of the altitude on side* BC, *then* $(AB)^2=(BC)^2+(CA)^2-2(BC)(DC)$. Since $DC=CA\cos BCA$, we recognize this last statement as essentially the so-called *law of cosines*, and the law of cosines is indeed a fine generalization of the Pythagorean Theorem.

But perhaps the most remarkable extension of the Pythagorean Theorem that dates back to the days of Greek antiquity is that given by Pappus of Alexandria at the start of Book IV of his *Mathematical Collection*. Pappus, the last of the great Greek mathematicians, flourished toward the end of the third century A.D., and with high enthusiasm and marked competence tried to rekindle fresh interest in the languishing Greek mathematics. Although Pappus wrote a number of commentaries on important Greek works of mathematics, his really great contribution is his *Mathematical Collection*, a combined commentary and guidebook to the existing geometrical works of his time. This work of Pappus is sown with numerous original propositions, improvements, extensions, and historical remarks, and it has proven to be a veritable mine of rich geometrical nuggets.

The Pappus extension of the Pythagorean Theorem is as follows (see Fig. 2): *Let* ABC *be* any *triangle and* CADE, CBFG any *parallelograms described externally on sides* CA *and* CB. *Let* DE *and* FG *meet in* H *and draw* AL *and* BM *equal and parallel to* HC. *Then the area of parallelogram* ABML *is equal to the sum of the areas of parallelograms* CADE *and* CBFG. The proof is easy, for we have $CADE=CAUH=SLAR$ and $CBFG=CBVH=SMBR$. Hence $CADE+CBFG=SLAR+SMBR=ABML$. It is to be noted that the Pythagorean Theorem has been generalized in two directions, for the right triangle of the Pythagorean Theorem has been replaced by *any* triangle, and the squares on the legs of the right triangle have been replaced by *any* parallelograms.

The student of high school geometry can hardly fail to be interested in the

Figure 2



Pappus extension of the Pythagorean Theorem, and the proof of the extension

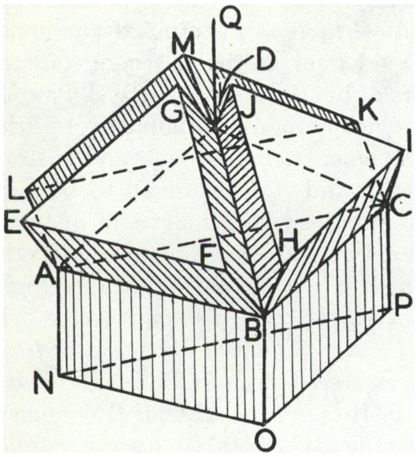


Figure 3

can serve as a nice exercise for the student. Perhaps the more gifted student of geometry might like to try his hand at establishing the further extension (to three spaces) of the Pappus extension: *Let* ABCD (see Fig. 3) *be any tetrahedron and let* ABD-EFG, BCD-HIJ, CAD-KLM *be any three triangular prisms described externally on the faces* ABD, BCD, CAD *of* ABCD. *Let* Q *be the point of intersection of the planes* EFG, HIJ, KLM, *and let* ABC-NOP *be the triangular prism whose edges* AN, BO, CP *are translates of the vector* QD. *Then the volume of* ABC-NOP *is equal to the sum of the volumes of* ABD-EFG, BCD-HIJ, CAD-KLM. A proof analogous to the one given above for the Pappus extension can be supplied.

What's new?

BOOKS

College

Mathematics in Business, Lloyd L. Lowenstein. New York: John Wiley and Sons, Inc., 1958. Cloth, xv+364 pp., $4.95.

Modern Business Statistics, John E. Freund and Frank J. Williams. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1958. Cloth, xv+539 pp., $10.00.

Modern Computing Methods. New York: Philosophical Library, 1958. Cloth, vi+130 pp., $8.75.

Problems in Euclidean Space: Application of Convexity, H. G. Eggleston. New York: Pergamon Press, 1957. Cloth, viii+165 pp., $6.50.

A Short Course in Differential Equations (2nd ed.), Earl D. Rainville. New York: The Macmillan Company, 1958. Cloth, x+259 pp., $4.50.

Teaching Arithmetic for Understanding (with teacher's manual and student handbook), John L. Marks, C. Richard Purdy, and Lucien B. Kinney. New York: McGraw-Hill Book Company, Inc., 1958. Cloth, xiv+429 pp., $6.00.

The Theory of Functions of a Real Variable and *The Theory of Fourier's Series* (Dover republication), E. W. Hobson. New York: Dover Publications, 1958. Paper, $6.00 a set.

The Theory of Groups (2nd ed.), Hans J. Zassenhaus. New York: Chelsea Publishing Company, 1958. Cloth, x+266 pp., $6.00.

A Treatise on Plane and Advanced Trigonometry (Dover republication), E. W. Hobson. New York: Dover Publications, Inc., 1957. Paper, xv+383 pp., $1.95.

Understanding and Teaching Arithmetic in the Elementary School, E. T. McSwain and Ralph J. Cooke. New York: Henry Holt and Company, 1958. Cloth, xi+420 pp., $5.50.

Miscellaneous

Communication, Organization, and Science, Jerome Rothstein. Indian Hills, Colorado: The Falcon's Wing Press, 1958. Cloth, xcvi+110 pp., $3.50.

Elements of Mathematical Biology (Dover republication), Alfred J. Lotka. New York: Dover Publications, Inc., 1956. Paper, xxx+465 pp., $2.45.

Fantasia Mathematica, edited by Clifton Fadiman. New York: Simon and Schuster, 1958. Cloth, xix+298 pp., $4.95.

Figurets, J. A. H. Hunter. New York: Oxford University Press, 1958. Cloth, x+116 pp., $3.50.

The Golden Number, M. Borissavlievitch. New York: Philosophical Library, 1958. Cloth, 91 pp., $4.75.